Part

3

Economic Evaluation and Public Policy

Chapter

6

Valuing Individual Preferences

Utility and its measurement lie at the heart of political economy.

Jules Dupuit

Individual Preferences and Utility ♦ Deriving Demand Curves from Preferences and Budget Constraints ♦ Valuation Principles Further Considered ♦ From Valuation Principles to Practice

Respect for individual values lies at the heart of democracy. And as Dupuit (1844), a French engineer, observed over 170 years ago, the valuation of individual preferences lies at the heart of the economic approach to public policy. What do people want? How are their preferences valued? In this chapter we discuss the concepts of preferences and how they can be valued in monetary units. It should be noted that monetary units are adopted as a practical measure. We could measure preferences in bottles of rum like the early British settlers, but dollars are much more practical. These measures of preference provide the fundamental elements from which public policy can be constructed.

As a starting point, we may suppose that the value of a good to someone is the price that he or she is willing to pay for it. More precisely it is the maximum price. This represents the value of other goods that he or she is willing to give up for the new good. This is generally viewed as a sensible approach. Nevertheless, complications arise and must be dealt with. First, the value of something is not unique; it depends on the quantity consumed. The value of the marginal item consumed may be a lot less than the average value of the good. Second, the price that someone is willing to pay for a good often depends on their income. If a person's real income changes, he or she may place a different value on a good. Third, there may be no observable price for some goods, notably for non-market goods.

In this chapter we discuss mainly conceptual issues in valuing goods. The first section describes the nature of preferences and the utility or wellbeing of individuals. We then discuss how to derive demand curves from preferences. The third main section discusses valuation principles in more detail. Finally, we discuss how to put these valuation principles into practice focussing on standard issues in estimating individual demand in markets. In Chapter 11 we discuss various practical issues of estimating individual preferences using other revealed or stated preference methods.

Individual Preferences and Utility

The standard economic theory of valuation starts from the premise that inferences about individual preferences can be drawn from observations of their choices in various contexts, but especially in markets. This is broadly known as revealed preferences. This assumes that individuals make rational (consistent and informed) choices and have sensible preferences. In Chapter 4 we saw that behavioural economists challenge these assumptions and have good reasons for doing so. However, this does not mean that the standard theory of revealed preference has no practical use. Bernheim and Rangel (2007) provide a comprehensive review of behavioural public economics and conclude that preferences may still be discovered by selective application of the revealed preference principle and that practising behavioural economics requires modifying rather abandoning the standard theory of revealed preference. In so far as preferences cannot be inferred from observed (revealed) choices, for whatever reason, other ways to value individual preferences may need to be adopted. Of these the most common way is the use of stated preference methods, which we discuss in Chapter 11.

To illustrate the nature of preferences it is useful to work with the choice between two goods, x and y. These goods can be consumed in various quantities, which make up a consumption bundle. This bundle can be represented by (q_x, q_y) where q_x and q_y denote the quantity of goods x and y respectively. Suppose that there are three consumption bundles (A, B and C) which are represented by (3, 4), (2, 5) and (4, 3) units of goods x and y respectively.

The theory of preferences assumes that individuals can rank all such bundles. This means that they either regard any two bundles as equal value (i.e. they are indifferent between them) or prefer one bundle to the other. Second, preferences are assumed to be transitive. Transitivity means that, given any three consumption bundles (such as A, B and C), if A is preferred to B and B is preferred to C, then A is preferred to C. Thirdly, if someone is indifferent between A and B and between B and C, she is indifferent between A and C. These assumptions, taken together, ensure that an individual has a complete preference ordering.

In addition, individuals are assumed to prefer more of any good. If bundle A contains more of one good and no less of the other than does bundle B, then bundle A is preferred. Finally, individuals are assumed to attempt to attain the highest level of satisfaction possible, which is the most preferred bundle of goods consistent with their budget. In other words, in their choices they attempt to maximise their utility subject to a budget constraint.

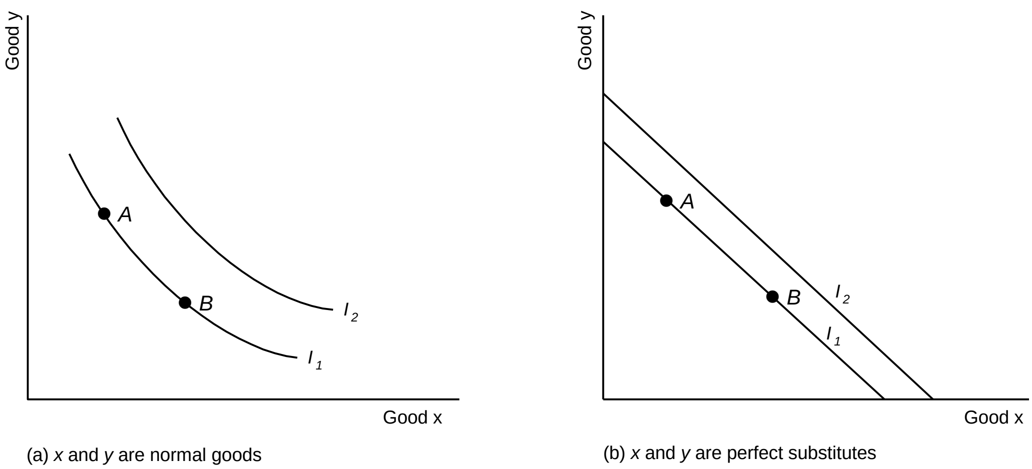

Figure 6.1 Alternative forms of indifference curves

Mapping preferences and trade-offs

These preference concepts can be illustrated with the aid of indifference curves (see Figure 6.1). An indifference curve shows combinations of goods that give an individual equal satisfaction (utility). Indifference curves imply trade-offs between goods and implicit values of goods. An extra unit of good x is worth the quantity of good y that an individual is willing to give up for it. A higher indifference curve implies a higher level of utility.

Indifference curves generally slope downwards and are usually convex. The downward slope shows that a person is willing to sacrifice some amount of one good in order to obtain more of the other. The slope of the curve at any point shows the marginal rate of substitution of good y for good x (MRS_{yx}). MRS_{yx} is the marginal amount of good y that a consumer is willing to sacrifice to obtain a unit increase in good x. A convex curve implies diminishing marginal rate of substitution. This means that the more units of good x that someone possesses, the less of good y he or she is willing to sacrifice to obtain an additional unit of good x. Point A in Figure 6.1a corresponds to a consumption bundle with a large amount of good y. The bundle at point B contains less of good y but more of x. Accordingly the slope of the indifference curve is flatter at point B than at point A. MRS_{yx} declines as we move down the indifference curve.

By contrast Figure 6.1b shows an indifference curve with a constant slope. In this case the two goods are perfect substitutes.

Various examples

Although the theory of preferences is usually illustrated by comparing bundles of market goods, such as clothes and food, the theory is general. Individuals can have preferences over market and non-market goods, over market goods (income) and leisure, and over current and future consumption (as we saw in Chapter 5). Moreover, each set of preferences can be represented by indifference curves.

Figure 6.2a shows the trade-offs that an individual would be willing to make to obtain more of an environmental good (such as air quality). To obtain a non-marginal increase of an environmental good from E_2 to E_3, the individual is willing to give up ($M_2 - M_3$) market goods. The convex indifference curve implies that as the quality of the environment increases, the individual is willing to sacrifice fewer market goods for additional units of the environmental good.

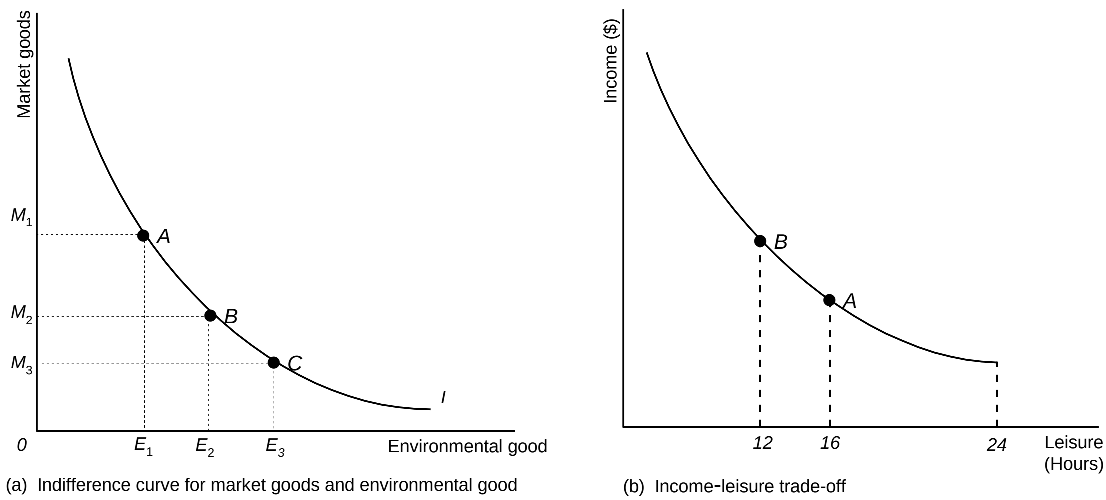

Figure 6.2 Indifference curves for other trade-offs

Figure 6.2b shows the trade-offs that an individual is willing to make between income and leisure. The slope of the indifference curve at any point shows the marginal rate at which individuals require to be compensated for loss of leisure. The slope generally increases as leisure declines. The less leisure that someone has, generally the greater will be the compensation required to give up an extra hour of leisure.

Utility functions

As we will see, a demand curve can be derived from an indifference map. However, the process of estimating demand is more precise and rigorous if mathematics is employed to describe consumer preferences and the indifference curves that depict these preferences. Economists use utility functions to represent these preferences in mathematical form.

Utility function
Shows an individual's utility as a function of their consumption of goods or their income and sometimes also their leisure

A **utility function** shows utility as a function of an individual's consumption of goods. Higher utility numbers indicate greater utility. However, the utility numbers are arbitrary and ordinal rather than cardinal. A bundle of goods with a utility of 200 is preferred to a bundle with a utility value of 100, but it does not necessarily imply twice as much satisfaction.

Consider first a simple linear utility function for an individual, in which goods x and y are perfect substitutes as in Figure 6.1b.

$$U = u\,(q_x, q_y) = q_x + q_y \tag{6.1}$$

If we have the following four consumption bundles for the two goods, (2, 4), (2, 6), (4, 2) and (5, 10), the corresponding utility numbers are 6, 8, 6 and 15. These numbers indicate the preference rank of the bundles. Bundle 4 is the preferred bundle followed by bundle 2 and by bundles 1 and 3. The individual is indifferent between bundles 1 and 3. However, the numbers cannot be interpreted as actual utility magnitudes.

We now rank the same four consumption bundles where the two goods are not perfect substitutes and the principle of diminishing marginal rate of substitution applies. To illustrate this case, suppose that the utility function has a Cobb–Douglas form where the exponents sum to one and exponent α equals 0.7.

$$U(q_x, q_y) = q_x^{\alpha} q_y^{1-\alpha} = q_x^{0.7} q_y^{0.3} \tag{6.2}$$

The estimated utility numbers for the bundles are now 2.56, 2.78, 3.87 and 6.16 respectively. Bundle 4 is still preferred, but followed by bundles 3, 2 and 1 respectively.

Deriving Demand Curves from Preferences and Budget Constraints

To derive demand curves that show what individuals are willing to pay for extra units of a good, we need to know their income (budget constraint) as well as their preferences. Assuming a one-period model in which an individual consumes all his or her income, the budget constraint is given by:

$$M = p_x q_x + p_y q_y \tag{6.3}$$

where M is the income of the individual and p_x and p_y are the prices of goods x and y respectively. The budget line that represents this constraint is shown along with a set of indifference curves in Figure 6.3 overleaf.

The budget equation can be rearranged as:

$$q_y = M/p_y - (p_x/p_y)\, q_x \tag{6.4}$$

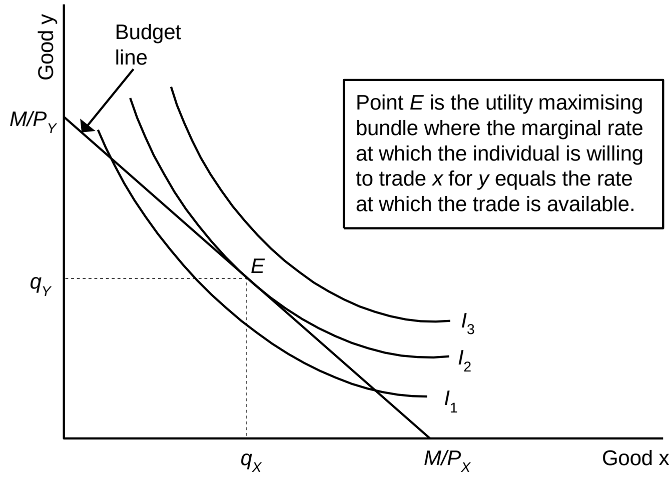

Figure 6.3 The consumption bundle that maximises utility

In this equation, M/p_y is the intercept term for good y and p_x/p_y is the slope of the budget line. Demand for y depends on real income (M/p_y) and the relative price at which a consumer can substitute good y for good x (p_x/p_y).

To know how much someone can obtain from their income, M must be divided by the prices of goods. Also, to know the real cost of something, we need to know relative prices. Suppose that p_x is \$2 and p_y is \$10; the relative price is 0.2. The real cost of good x is 0.2_y. If p_y falls to \$2, the real cost of good x is 1.0_y. The rearranged budget equation (6.4) implies that economic behaviour depends on real income and relative prices, not on nominal magnitudes.

The slope of the budget line is the ratio of the prices of the two goods (the rate at which an individual can trade good y for good x). If we combine this rate with the MRS (the marginal rate at which an individual is willing to trade) we obtain the consumption bundle that a utility-maximising consumer chooses. The assumptions that a consumer has downward-sloping, convex indifference curves and prefers more to less imply that the utility-maximising consumption bundle must lie on the budget line. As shown in Figure 6.3, the utility-maximising bundle is the tangent point (E) of the indifference curve and the budget line. At this point, the MRS equals the relative price.

Demand curves

Figure 6.4 shows the quantity of good x demanded at various prices. To estimate the demand, we shift the budget line to reflect the change in p_x and estimate the new utility-maximising consumption bundles (E, F and G). For simplicity, indifference curves are not shown in this figure. Note that M and p_y are held constant so that the y-intercept is constant. The price consumption curve shows the utility-maximising quantities of good x at each price for x (\$6, \$4 and \$2). The relevant quantities (20, 30 and 40 respectively) can be read off the graph.

Formally, the individual's choice of consumption bundle is a constrained maximisation problem. The consumer chooses quantities of x and y (q_x and q_y) that maximise their utility $U(q_x, q_y)$ subject to their budget constraint ($M = p_x.q_x + p_y.q_y$). The outcome is:

$$q_x^* = D_x\ (p_x, p_y, M) \qquad (6.5)$$

$$q_y^* = D_y\ (p_x, p_y, M) \qquad (6.6)$$

where q_x^* and q_y^* are the utility-maximising quantities, which can be represented as demand functions D_x and D_y.

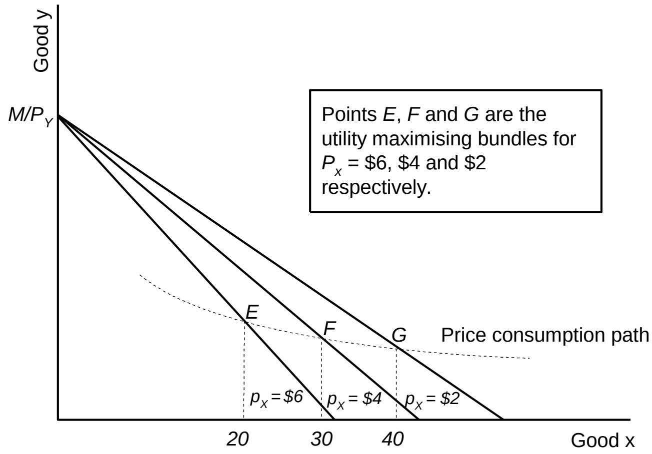

Figure 6.4 Estimating demand from price consumption path

Box 6.1 Estimating a demand function from a specified utility function

Following the notation in the text, suppose that the utility function for an individual takes on the Cobb–Douglas form:

$$U(q_x, q_y) = q_x^{0.7} q_y^{0.3}$$

The budget constraint is given by:

$$M = p_x q_x + p_y q_y$$

The Lagrangian function is formed by combining the utility function and budget constraint:

$$L = q_x^{0.7} + q_y^{0.3} + \lambda(M - p_x q_x - p_y q_y)$$

Differentiating L with respect to q_x, q_y and λ and setting the partial derivatives to zero:

$$dL / dq_x = 0.7 q_x^{-0.3} q_y^{0.3} - \lambda p_x = 0$$
$$dL / dq_y = 0.3 q_x^{0.7} q_y^{-0.7} - \lambda p_y = 0$$
$$dL / d\lambda = M - p_x q_x - p_y q_y = 0$$

Solving the equations for q_x and q_y we have the following utility-maximising quantities:

$$q_x^* = 0.7M/p_x$$
$$q_y^* = 0.3M/p_y$$

With this utility function, expenditure shares are constant. Also, the demand for x does not vary with p_y and the demand for y does not vary with p_x. This feature of the Cobb–Douglas function limits its suitability for applied work.

Thus, the optimal quantities depend on the prices of goods x and y and money income. Different preferences lead to different demand functions. Footnote 1 shows how these general results are derived.[1] Box 6.1 shows how specific demand functions can be derived from a particular (Cobb–Douglas) utility function.

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[1] When the utility functions are differentiable, we can use the Lagrange multiplier method to solve for the demand functions. A Lagrangian function is formed by combining the utility function, budget constraint and the Lagrange multiplier (λ):

$$L = U(q_x, q_y) + \lambda(M - p_x q_x - p_y q_y)$$

By differentiating the Lagrangian function with respect to qx, qy and λ and setting the partial derivatives to zero, we obtain a system of three simultaneous equations:

$$U_x(q_x^*, q_y^*) - \lambda p_x^* = 0$$
$$U_y(q_x^*, q_y^*) - \lambda p_y^* = 0$$
$$M - p_x x^* - p_y y^* = 0$$

Demand functions for good x and y can then be obtained by solving for the unknowns q_x^*, q_y^* in terms of p_x, p_y and M.

Substitution and income effects

Substitution effect
The change in consumption of a good or service due to a change in relative prices

With ordinary demand curves, price changes have two effects: substitution and income effects. The **substitution effect** is the effect of a price change on demand due to the change in relative prices, holding the consumer's real income (utility) unchanged. Holding p_y and M constant, if p_x falls, the relative price of good x falls; if p_x rises, good x becomes relatively more expensive. Assuming a diminishing marginal rate of substitution, the substitution effect is always negative. That is, consumption increases as relative price falls and vice versa.

Income effect
The change in consumption due to a change in real income

The **income effect** of a price change is the change in demand due to a change in the real income of consumers. Holding p_y and M constant, a change in p_x changes a consumer's real income. If p_x falls, the budget set expands and the consumer ends up on a higher indifference curve (real income increases). If p_x rises, the budget set contracts and the consumer ends up on a lower indifference curve (real income falls). For normal goods, the income effects of price changes reinforce the substitution effects.

Substitution and income effects are illustrated in Figure 6.5. Amy's initial budget line for two goods, food and clothes, is shown by line AB. Given her indifference curve I_1, Amy maximises her utility at point E_1 and consumes Y_1 units of food and X_1 units of clothes. After the price of clothes falls, the budget line rotates to AC. Amy chooses point E_2 on indifference curve I_2 and consumes Y_2 and X_2 units of food and clothes respectively. To decompose this change in consumption into substitution and income effects, we draw a price line ($A'C'$) parallel to the new set of relative prices and at a tangent to the initial utility curve (I_1). Holding utility (real income) constant, the increase in consumption of clothes from X_1 to X_3 is the substitution effect due to the change in relative prices. The increase from X_3 to X_2 is the income effect.

Ordinary demand curve
A demand curve that shows how quantity demanded varies with price, holding money income constant. Also called observed demand curve

Ordinary and compensated demand curves

Thus, there are two kinds of demand curve. **The ordinary demand curve** (known as the Marshallian demand curve) shows the quantities demanded at all prices holding nominal income constant. This demand curve is derived from the price consumption curve as in Figure 6.4. This curve includes changes in real income. The ordinary demand curve is also called the observed demand curve.

Compensated demand curve
A demand curve that shows how quantity demanded varies with price, holding real income (utility) constant

The compensated demand curve (known as the Hicksian demand curve) shows the effect of price changes on the quantity demanded, holding real income constant (i.e. for a constant utility level). It is derived by varying the price changes as in Figure 6.5 and by drawing out the demand curve associated only with the substitution effects, excluding changes in real income. We cannot hold utility constant when prices change and money income is fixed. With the compensated demand curve, the consumer gains or loses notional income to hold their utility constant. This eliminates the income effect of a price change.

A typical relationship between an ordinary and a compensated demand curve is shown in Figure 6.6. Suppose that the two curves intersect at initial price P_1 and consumption of Q_1. With the ordinary demand curve, if the price falls to P_2, consumption increases to Q_2. If the price rises to P_3, consumption falls to Q_3. Both changes include income effects. The compensated demand curve excludes income effects and is steeper for a normal good. If price falls to P_2, consumption increases only to Q_4. If the price rises to P_3, consumption falls to Q_5.

The expenditure function. The problem confronting the utility-maximising consumer can also be viewed as an expenditure-minimising problem. In this case, the aim is to find the minimum expenditure necessary to achieve a specified level of utility. Instead of moving along the budget line until the highest indifference curve is reached, the consumer now moves along their indifference curve until the lowest iso-expenditure line is reached.[2] This shows the minimum level of expenditure necessary to achieve a given utility level as a function of prices and the required utility level. We draw on expenditure functions below to estimate the effects of changes on individuals. Box 6.2 overleaf derives compensated demand functions for a

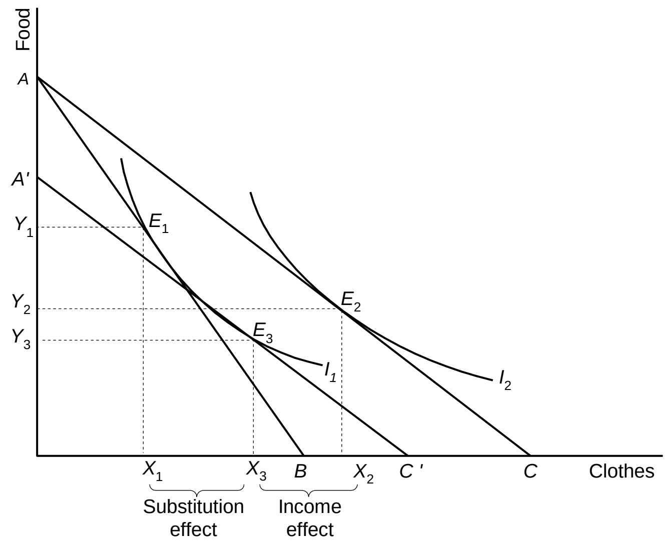

Figure 6.5 Income and substitution effects of a price decrease

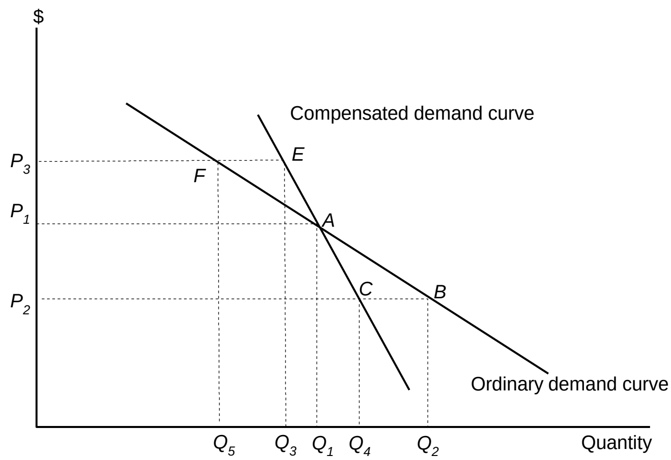

Figure 6.6 Ordinary and compensated demand curves

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[2] The isoexpenditure line is similar to the budget line and satisfies the equation $p_x\, q_x + p_y\, q_y = M^0$ (M^0 is a fixed expenditure). A Lagrangian function can be written for this minimisation problem as: $L = p_x q_x + p_y q_y + \mu(U^0 - U(q_x, q_y))$. The general compensated demand functions for x and y are: $q_x^{CD} = CD_x(p_x, p_y, U^0)$ and $q_y^{CD} = CD_y(p_x, p_y, U^0)$. Substituting these optimal values in $p_x q_x + p_y q_y$ gives:

$$(p_x.CD_x(p_x, p_y, U^0) + p_y.CD_y(p_x, p_y, U^0) = M(p_x, p_y, U^0),$$

which is called the expenditure function particular optimisation problem. Box 6.3 shows how income and substitution effects can be estimated,

As we have seen, the demand for a good depends on whether real income changes as with an ordinary demand curve or held constant as with a compensated demand curve. We need therefore to define more precisely the price that individuals are willing to pay for a good. This will also lead us to the question of what individuals are willing to accept as compensation for not having some good or service.

To do this we introduce the concepts of compensating variation (CV) and equivalent variation (EV) measures of value. We then discuss valuation with an ordinary demand curve which we will call the consumer surplus (CS) method. Finally we discuss the relationships between CV, EV and CS measures of value and how to choose between them.

Compensating and equivalent variations

The difference between CV and EV values is the reference point. For CV, the reference point is the individual's current level of utility *before* an economic change; for EV, the reference point is his or her level of utility *after* the change.

Box 6.2 Estimating compensated demand functions

Let the utility function and budget equation be as in Box 6.1. Consider a situation in which the government subsidises consumers so as to leave their utility unchanged. Assume this is done by a lump-sum payment which gives the consumer the minimum income necessary to achieve their initial utility level. Their compensated demand function will give the quantities of commodities that they will buy as functions of commodity prices under these conditions. This can be obtained by minimising consumers' expenditures subject to the constraint that their utility is at a fixed level U^0. The Lagrangian function in this case is:

$$L = p_x q_x + p_y q_y + \mu(U^0 - q_x^{0.7}\, q_y^{0.3})$$

Differentiating L with respect to q_x, q_y and μ and setting the partial derivatives to zero:

$$dL / dq_x = p_x{}^{-0.7} q_x{}^{-0.3} q_y{}^{0.3} \mu = 0$$
$$dL / dq_y = p_y{}^{-0.3} q_y{}^{-0.7} q_x{}^{0.7} \mu = 0$$
$$dL / d\mu = U^0 - q_x{}^{0.7} q_y{}^{0.3} = 0$$

Solving the equations for q_x and q_y, we have compensated demand functions:

$$q_x{}^{CD} = U^0 / \left(\frac{3}{7}\frac{p_x}{p_y}\right)^{0.3}$$

$$q_y{}^{CD} = U^0 / \left(\frac{7}{3}\frac{p_y}{p_x}\right)^{0.7}$$

Box 6.3 Substitution and income effects

The expenditure function is given by: $M(p_x, p_y, U^0)$.

Therefore by definition:

$$CD_x(p_x, p_y, U^0) = D_x[p_x, p_y, M(p_x, p_y, U^0)]$$

If we consider a price change of x, that is differentiating with respect to p_x, we get:

$$\frac{\partial CD_x}{\partial p_x} = \frac{\partial D_x}{\partial p_x} + \frac{\partial D_x}{\partial M}\frac{\partial M}{\partial p_x}$$

Rearranging:

$$\frac{\partial D_x}{\partial p_x} = \frac{\partial CD_x}{\partial p_x} - \frac{\partial D_x}{\partial M}\frac{\partial M}{\partial p_x}$$

The above equation shows the effect of a price change on the ordinary demand curve as substitution effect (the first part of the right-hand side shows the effect of a price change on the quantity demanded when utility level is fixed) and income effect (the second term on the right-hand side shows the effect of a price change on demand through the change in purchasing power).

Given that: $$\frac{\partial M}{\partial p_x} = q_x$$

The income effect is: $$-q_x \frac{\partial q_x}{\partial M}$$

which is negative for a normal good (because $\frac{\partial q_x}{\partial M} > 0$ for a normal good).

Consider first the **compensating variation**:

- The CV value of a good is the maximum amount that an individual would be willing to pay (WTP) for it and be no worse off with it than without it. This equals the income that can be taken away from someone and leave him or her at their initial utility level.
- The CV measure for loss of a good is the minimum amount that an individual would be willing to accept (WTA) as compensation for the loss and be no worse off than before.

Compensating variation
The amount of money to be given to, or taken from, an individual that would restore him/her to his/her initial level of utility following a change in price or consumption

Turning to the **equivalent variation**:

- The EV value of a good is the minimum amount that an individual would be willing to accept (WTA) and be as well off without the new good as he or she would be with it.
- The EV measure for loss of a good is the maximum amount that someone would be willing to pay to avoid the loss given that it would otherwise occur.

Equivalent variation
The amount of money to be given to, or taken from, an individual that would allow him/her to experience the same level of utility that he/she would experience with a given change in price or consumption

To estimate the CV value of a good, suppose that an individual has an initial level of utility of U_0 with a money income Y_0 and an amount of a public good G_0:

$$U_0\ (Y_0,\ G_0) \qquad (6.7)$$

Suppose that government proposes to increase the amount of the public good to G_1, which would increase the individual's utility to U_1:

$$U_1\ (Y_0,\ G_1) \qquad (6.8)$$

We want to know by how much the extra amount of the public good increases the individual's utility but we cannot directly measure U_0 or U_1. We therefore seek an indirect measure of the benefit by estimating what an individual would be WTP for the increase in the public good but remain on the initial level of well-being.

$$U_0\ (Y_0 - WTP,\ G_1) = U_0\ (Y_0, G_0) \qquad (6.9)$$

This WTP amount is the (CV) monetary value of the benefit from the increase in the public good from G_0 to G_1.

To estimate the EV value of a good, we ask how much income an individual would be WTA to forgo the increase in the public good and still be as well off as if he or she had received the extra amount of the public good. In this case we consider the combinations of money income and public good that would yield an equal level of utility (U_1).

$$U_1\ (Y_0 + WTA,\ G_0), = U_1\ (Y_0, G_1) \qquad (6.10)$$

Similar measures can be derived for losses in utility. However, in this case, the CV amount is measured by WTA and the EV amount by WTP. Suppose that the change from G_0 to G_1 denotes a loss of some amount of a public good. The CV is the amount of money that will compensate the individual for the loss and leave him or her at their initial level of utility.

$$U_0\ (Y_0 + WTA,\ G_1) = U_0\ (Y_0, G_0) \qquad (6.11)$$

The EV is the amount of money that an individual would be WTP to avoid the change.

$$U_1\ (Y_0 - WTP),\ G_0) = U_1\ (Y_0, G_1) \qquad (6.12)$$

The concepts of CV and EV are illustrated below using the example of (1) a new railway line and (2) a reduction in fares for an existing rail service.

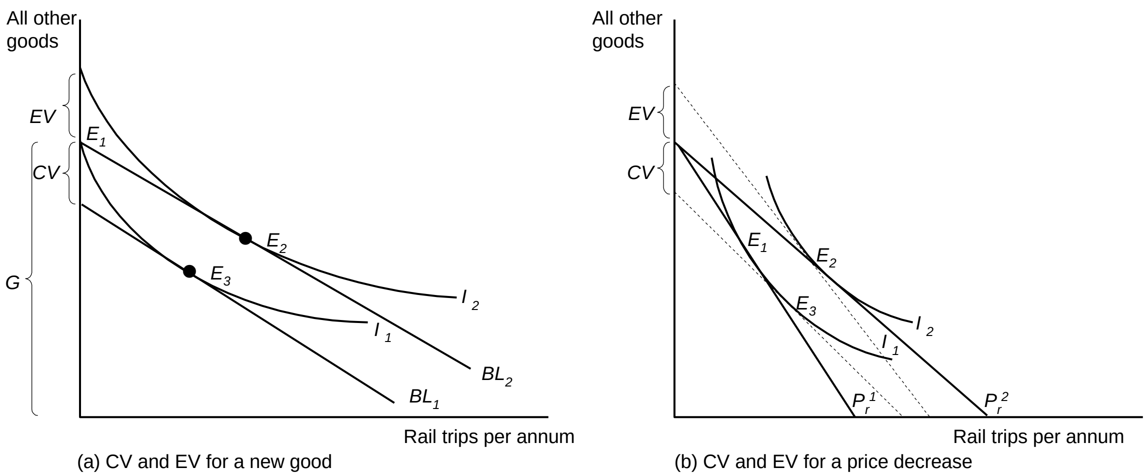


Figure 6.7 Compensating and equivalent variations

Valuations for a new good. Figure 6.7a shows Amy's consumption bundle without and with a railway line. Without a rail line, Amy is at equilibrium point E_1. She consumes G units of all other goods at an average composite price p_g and achieves indifference curve I_1. With a new rail line, given a price (p_r) for a rail trip, her new budget line is shown as BL_2, with a slope of p_r / p_g. Her new equilibrium point is E_2 and her utility has risen to the I_2 curve.

To estimate CV, we draw a budget line BL_1, with a slope of p_r / p_g. Given the indifference curve I_1, the individual would choose equilibrium point E_3. This indicates that, if the rail is built and \$CV are taken away from Amy, she will just be as well off as without the rail line because she is on her initial indifference curve I_1 in both cases. Thus, CV represents the maximum price that Amy will pay for the new rail line given the rail price.

On the other hand, taking the new utility level I_2 as the reference point, the equivalent variation is shown by EV. This is the extra income that would make Amy as well off without the new rail as she would be with it.

Valuations of a price reduction. Now suppose that the rail agency reduces the rail fare from p_r^1 to p_r^2 and the price of other goods p_g does not change. As shown in Figure 6.7b, the CV is the amount of money that can be taken from an individual in the new lower price situation and leave her as well off (at utility level I_1) as with the initial higher price. The benefit of the fall in price is shown by the distance CV. In terms of the consumer expenditure function:

$$CV = M(p_r^1, p_g, I_1) - M(p_r^2, p_g, I_1) \tag{6.13}$$

The EV of the price change is the amount of money that must be given to an individual in the initial price situation to make her as well off as she would be with the new lower price. Here the new utility level (I_2) is the reference point. The equivalent variation for the fall in real fare is shown as the distance EV in Figure 6.7b. In terms of the consumer expenditure function:

$$EV = M(p_r^1, p_g, I_2) - M(p_r^2, p_g, I_2) \tag{6.14}$$

Comparing CV and EV

Of course, the distinction between CV and EV is of no consequence when they produce the same result. If the marginal value of a dollar of consumption is constant over changes in income, CV = EV. In this case, utility is a linear function of income.

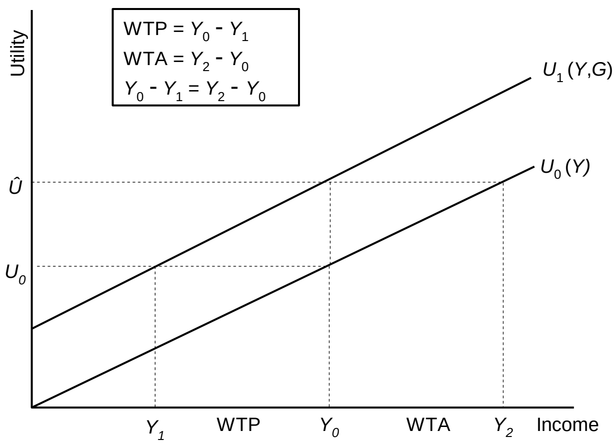
Figure 6.8a shows two linear utility functions. The U_0 curve shows utility simply as a function of private income. The U_1 curve shows utility as a function of private income and a public good. For simplicity we assume here that a public good is either provided or not. We are not dealing with different quantities of a public good. Suppose that an individual has a private income of Y_0 and no public good. She has a utility of U_0. If the public good were provided, she would be willing to pay $Y_0 - Y_1$ for the good and remain at U_0. This is the CV valuation of benefit. On the other hand, if she is deemed to have a right to the public good she would be entitled to utility $\hat{U}$. She would be willing to accept $Y_2 - Y_0$ income for loss of the public good and remain at $\hat{U}$ level of well-being. This may be interpreted as the EV value of the good. However, with linear utility curves, $Y_0 - Y_1 = Y_2 - Y_0$ so that WTP = WTA. In this case the CV valuation = the EV valuation.

This result does not hold if utility is a non-linear function of income. Utility is generally expected to rise with income but at a declining rate as an extra dollar to a rich person has less value than a dollar to a poor one. This is shown in Figure 6.8b. Using the same terms as in Figure 6.8a, it can now be seen that $(Y_2 - Y_0) > (Y_0 - Y_1)$. In words, the WTA compensation for not having the good is greater than the WTP to obtain it. In this case the EV value for a good would be greater than the CV value.

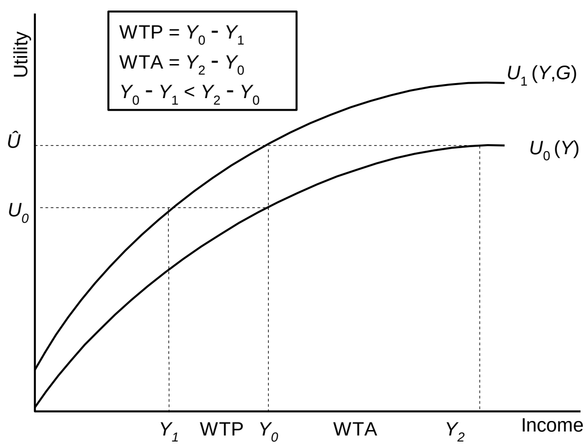
We now turn to valuing the loss of a good again assuming diminishing marginal utility as in Figure 6.8b. In this case the starting point includes the public good, which is $\hat{U}$ on the U_1 curve. The individual is here assumed to have a right to the public good. The WTA amount is again greater than the WTP amount, $(Y_2 - Y_0) > (Y_0 - Y_1)$. However, in this case $(Y_2 - Y_0)$ is the CV value and $(Y_0 - Y_1)$ is the EV value and the CV value is greater than the EV value.

Ordinary (observed) demand and consumer surplus

An ordinary demand curve can be viewed as both a marginal WTP schedule and as a marginal benefit schedule. To see this, consider Figure 6.9a. This figure shows, in a series of discrete blocks, a demand schedule for electricity in terms of kilowatt hours (kWh) per week. The consumer values the first kWh at 90 cents, the second unit at 80 cents, and so on down to the fortieth unit at 9 cents per kWh. These amounts represent marginal benefits.

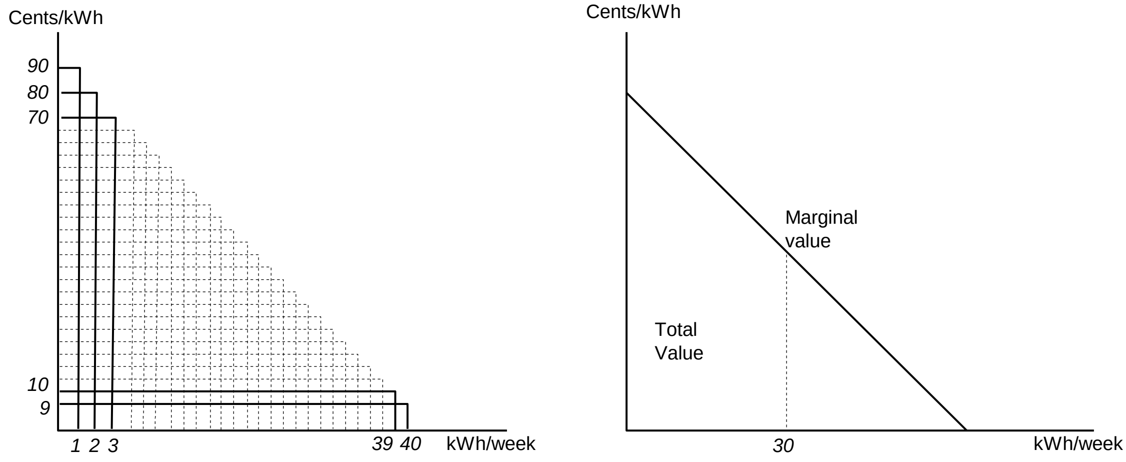


(a) WTP for a good equals WTA for not having it



(b) WTP for a good is less than WTA for not having it

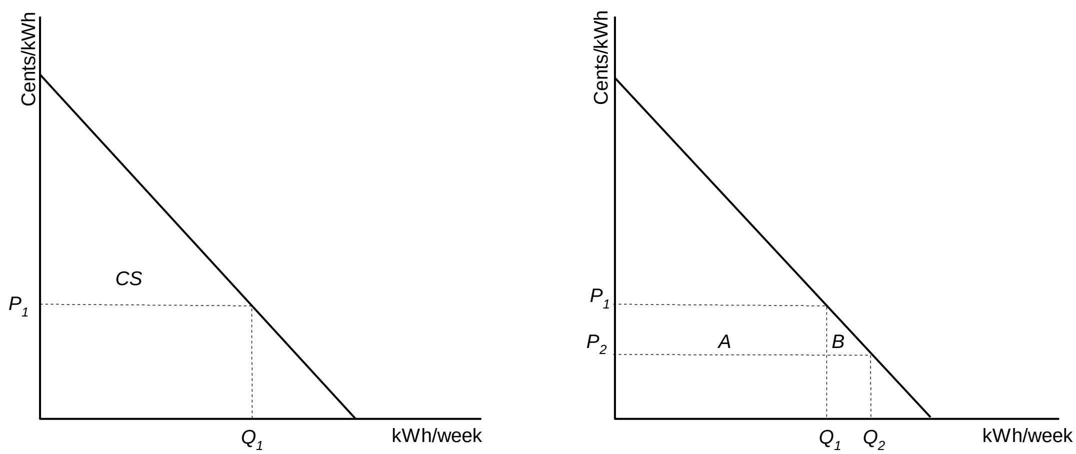
Figure 6.8 Comparison of willingness to pay and willingness to accept



(a) Marginal unit values

(b) Total and marginal value

Figure 6.9 Valuing electricity via a demand curve



(a) Consumer surplus for a new good

(b) Consumer surplus for a price change

Figure 6.10 Valuation and consumer surplus

Typically, a demand curve is drawn as a continuous downward-sloping curve as in Figure 6.9b. The vertical distance from the demand curve to the horizontal axis is the marginal value of the respective unit. The area under the demand curve represents the total value of electricity consumed.

Consumer surplus
The excess of the benefit that an individual gains from purchase of a good over the amount paid for it

Consumer surplus (CS) is the difference between the maximum amount that an individual is willing to pay for a good and its price. Figure 6.10a shows a consumer's surplus for consumption of electricity. The total CS is the area between her demand curve up to the amount consumed and the market price. Figure 6.10b shows the increase in CS (areas $A + B$) when the price of electricity is reduced from P_1 to P_2. Area A is the gain related to existing consumption (Q_1), which is Q_1 ($P_1 - P_2$). Area B is the surplus associated with the increase in consumption. The consumer gains a large surplus of almost ($P_1 - P_2$) for her first extra units of consumption but a very small surplus for her last additional units of consumption. When the demand curve can be represented by a straight line, area *B* equals 0.5 $(Q_2 - Q_1)(P_1 - P_2)$. Of course, if the price rises from P_2 to P_1, there would be an equivalent loss of consumer surplus.

Using the change in CS (ΔCS) with an ordinary demand curve as a measure of value, the value of a beneficial change is the maximum amount that an individual is willing to pay to move from their initial level of utility to a higher level. The cost of an adverse change is the maximum amount that he or she is willing to pay to avoid a fall to a lower level of utility. These are payments to achieve, or to avoid, changes in utility levels. They are subtly different concepts to CV or EV payments that would leave an individual on an existing or new utility level respectively.

Consumer surplus, CV and EV

Whereas CV and EV measures hold utility constant, an individual's utility changes as he or she moves along a demand curve. These differences are illustrated in Figure 6.11. Assume an initial price and quantity position of P_0 and Q_0 respectively. The price falls to P_1 and quantity consumed rises to Q_1. The CS is the area between the demand curve and the P_1 line, which equals area (*A* + *B*). The CV demand curve holds real income constant at the initial position. This eliminates the income effect and the CV measure of benefit is area *A*. On the other hand, the EV demand curve holds real income constant at the P_1 Q_1 position. In this case the WTA value for not having the price fall is given by area (*A* + *B* + *C*). If the starting point is the P_1 Q_1 position, the CV and EV curves are reversed.

In summary, CV, EV and CS are equal if the marginal utility of income is constant, as in Figure 6.8a. Equivalently CV = EV if the marginal rates of substitution between public and private goods are constant (if utility is a linear function of different levels of the two goods).

However, when there is a significant change in either (1) real income and the marginal utility of consumption or (2) the relative supply of goods, CV, EV and CS may not be equal.

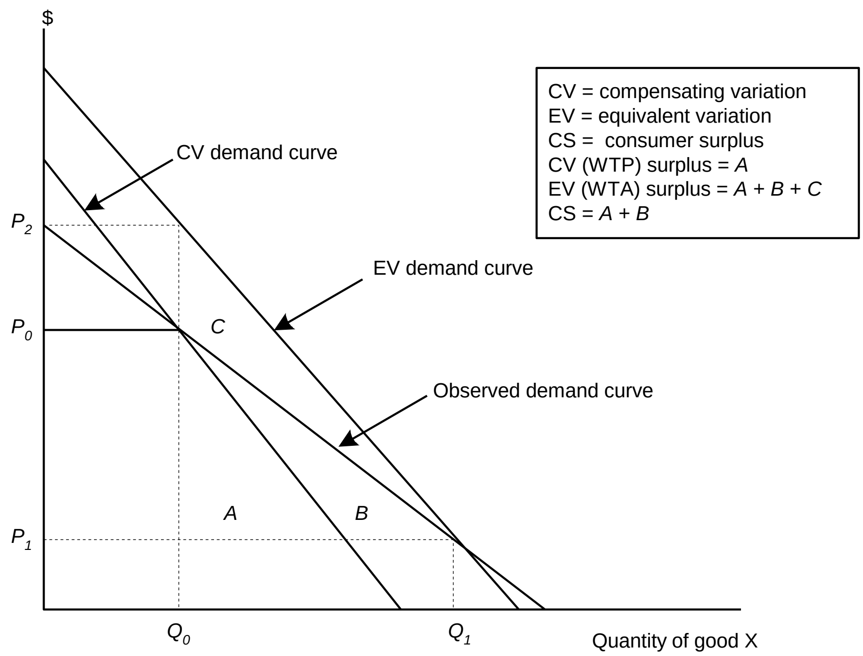


Figure 6.11 Benefits of a fall in price: summarising valuation differences

Table 6.1 Comparison of valuation measures

Income effects	*Beneficial change* *New good or price fall*	*Adverse change* *Loss of good or price rise*
Income effects occur	CV < ΔCS < EV	CV > ΔCS > EV
No income effects	CV = ΔCS = EV	CV = ΔCS = EV

The qualitative relationships between CS, CV and EV measures of value for beneficial and adverse changes are summarised in Table 6.1. When there are no real income effects, the three measures are equivalent. When there are real income effects, CV gives the lowest measure of value for a beneficial change and the highest value for an adverse change. Consumer surplus gives an intermediate value for both beneficial and adverse changes.

Given these potential differences between these three measures of value, which measure should be used? And which measure is most often used?

Choice of valuation principle

The choice of measure of value depends on views about individual rights. If a person has a right to a beneficial change, for example a right to a new hospital service or lower rail prices, the value of the services is the EV amount (the compensation required if the service or subsidy is not provided) rather than a CV or CS amount. On the other hand, if a person loses an existing good, a CV measure of value may be appropriate (the minimum amount that he or she would be WTA as compensation for the loss). Crucially, the choice of measure depends on ethical or political judgements rather than on technicalities.

Where CV and EV values differ considerably, the policy implications may be significant. The CV principle implies that the current situation is an appropriate reference point for policy evaluation and that changes from it must be justified. Suppose that government proposes to clean up a polluted river. Using the CV approach, we would estimate the maximum amounts that the community would be WTP for the clean river and be no better off than before. If the sum of these WTP amounts exceeds the clean-up costs, the river should be cleaned up. On the other hand, if we start from the position that the river should be clean, EV valuation principles apply. We would need to know what people were WTA as compensation for the polluted river. If the sum of WTA amounts exceeds the cost of cleaning up the river, the river should be cleaned up. This could give a quite different outcome.

Or suppose that government is considering whether to provide a higher level of hospital services in a large country town. Under the CV approach we would compare what residents would be WTP for these services and be no worse off with the costs of providing them. If WTP amounts exceeded the costs, the services would be provided. Under the EV approach we would compare what residents would be WTA for not having the services with the costs. The services would be provided if the WTA amounts exceeded the costs.

In practice, CS is the most commonly used valuation method for four reasons. First, it is based on observed demand curves. CV and EV amounts have to be estimated using hypothetical scenarios holding utility constant at the initial or post-change positions. This means either estimating compensated demand curves which take out income effects or running appropriate surveys. Thus, CS measures are generally the most practical measure of value. Estimates of WTA compensation values usually require surveys because these values are not often observed in markets. Second, CS values are often a close approximation to CV or EV values. Willig (1976) estimated that when income effects are small, the differences between WTP and WTA values are less than 5 per cent for most goods. Third, when there are significant income effects, the CS measure is a compromise between the CV and EV measures. Fourth, rights are often not clear. Individuals may have a right to clean air, but also a right to power supply and travel. Often the same individuals want all these goods. In such cases, it makes sense to estimate how much people are WTP for each potential use of the environment (essentially a CS measure) rather than to start with a preconception about how who owns environmental property and to weight the valuation procedure according to these presumptions.

However, the differences between the measures are not always minor. In a survey of 45 studies of WTP and WTA values, Horowitz and McConnell (2002) found that WTA values can be as much as 10 times WTP values for some health and safety goods or services and are on average seven times higher. The difference between WTA and WTP values rises with the size of the income effect and the income elasticity of demand for the good. It also rises with a scarcity of substitutes for the good. When a good is scarce, individuals require more compensation for its loss.[3]

Differences between WTA and WTP values may also reflect loss aversion as well as on views about rights. Loss aversion occurs when someone places a higher value on a good that is lost than on a gift of the same good, not because of decreasing marginal utility of consumption but because of a dislike of losses, or what Kahnemann and Tversky (1979) famously called the endowment effect. Also, individuals may claim high compensation for a good because they believe they have a right to it and for which they believe they should not have to pay.

Therefore, when individuals lose a good or a property right, a CV measure of value for loss of it may be appropriate. This would be the minimum WTA amount that an individual would accept as compensation for the loss. This is consistent with most popular notions of fairness and indeed the notion of property rights. On the other hand, when individuals have a right to a new good or service, the estimated EV value may be considered more appropriate than the respective CV or CS valuation. This would be the minimum WTA amount that they would accept for not receiving the good or service rather than what they would be WTP to obtain it.

From Valuation Principles to Practice

We now consider some issues in estimating these demand curves for market goods. We also discuss briefly below how to estimate the compensation that individuals are WTA for losing a good (CV values) or the compensation required for not receiving a good to which they have a perceived right (EV values). In Chapter 11 we discuss methods for valuing non-market goods.

Estimating ordinary demand curves

For some policy issues it may be sufficient to estimate only one point on the demand curve: the price that consumers are WTP a particular quantity of goods. However, for many valuations, estimates of the whole demand curve or major parts of it are required.

Many textbooks describe methods for estimating ordinary demand curves (see, for example, Gujarati, 2003). Here we note two major issues: the multivariate nature of the demand function and the identification problem of distinguishing demand and supply.

A demand curve is simply a relationship between the quantity of a good x demanded (Q_x^d) and its price (P_x). However, care must be taken in estimating a demand curve simply by regressing observations of Q_x against P_x because many factors may influence quantity demanded. To estimate the effects of price on demand, it is often necessary to estimate a general demand function that includes other variables such as income and population. For example, if a rail agency wishes to estimate the demand for a railway trips, the agency could estimate a multiple regression equation in which the number of rail trips in a specified period

[3] Hanemann (1991) shows that technically the difference between WTA and WTP depends on the ratio of the income effect to the substitution effect.

is regressed against such variables as rail fares, trip times and service frequency, bus fares, road travel speeds, household income, car ownership and populations at various distances to rail stations.

Evidently, estimating a demand function may require considerable data. Cross-sectional analysis usually provides a richer data set than time series analysis. However, cross-sectional analysis does not always provide data with differing prices that are critical to demand analysis. Time series analysis may provide more information on price changes and their effects.

The second main issue in estimating a demand function (the identification problem) is the problem of estimating the parameters of a structural equation when we observe equilibrium positions. Suppose we observe two prices for a good and the quantities purchased at each price. We may be observing two points on a demand curve or on a supply curve or two equilibrium points reflecting shifts in demand and supply. If we estimate the relationship between quantity and price, we need to know whether we are estimating a demand or a supply function. The identification problem may be resolved if we have additional variables in either the demand or the supply function that allow the two curves to be differentiated. If income is included in the demand function along with price, a change in income will cause the demand curve to shift and each shift in the demand curve creates a new intersection of demand and supply, essentially mapping out a supply curve. Conversely, if a variable measuring weather conditions is included in the supply function, a change in this variable would shift the supply function and the equilibrium points would indicate a demand curve. When demand and supply are determined simultaneously by price, more complex statistical methods, such as two-stage least squares, are required to estimate the demand or supply curve.

Fortunately, in the public sector the supply of services is often exogenous and independent of price and the observed quantity and price data can be assumed to represent a demand curve. However, specification of the demand curve as linear, log-linear or another functional form requires careful analysis and can affect estimates of demand and consumer surplus.

Finally, as a practical matter, when valuing changes to existing services, it is often possible to draw on price and income elasticities that have been estimated in research studies. For example, many research papers have shown that the price and cross-price elasticities for public transport services are usually low (Goodwin, 1992). For new goods, specific market research and econometric work may be required.

Estimating consumer surplus and compensating and equivalent variations

Most econometric estimates of price effects include substitution and real income changes and are thus estimates of consumer surplus. Estimating compensated demand curves, or CV and EV amounts, is more complicated. To estimate a compensated demand curve the analyst must estimate how quantity demanded varies with price holding real income constant. Once the compensated demand curve is estimated, estimating the relevant area under the demand curve would be straightforward. However, we often lack sufficient data to estimate quantity demanded as a function of changes in prices holding income constant.

The following example shows how CS, CV and EV may be estimated in a specific case. Suppose that a household spends \$10 000 per annum, representing 25 per cent of its income, on renting housing. To convert this figure of \$10 000 into prices and quantities, suppose that the price of housing is \$100 per m^2 per annum and that the household rents 100 m^2. Government decides to subsidise rental housing by 10 per cent. Thus the cost of housing falls to \$90 per m^2 and, assuming a price elasticity of demand of –1.0, the household purchases 110 m^2 of housing. This is shown in Figure 6.12.

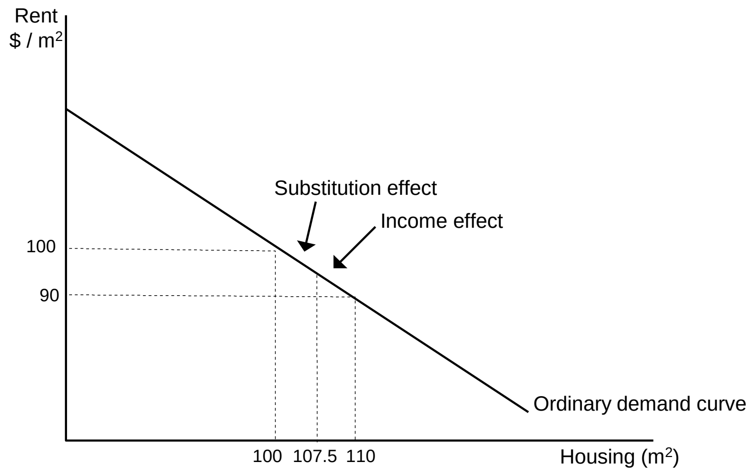


Figure 6.12 Estimating CS and CV for a price fall

Approximating the ordinary demand curve as a linear curve, the benefit to the household measured by the change in consumer surplus can be calculated as:

$$\Delta CS = (100 \times 10) + (10 \times 10 \times 0.5) = \$1050$$

To estimate the CV associated with the fall in price, we need to take out the income effect. Suppose that the income elasticity of demand for housing is 1.0. Because the household receives (initially) a subsidy of $1000 per annum, its real income has increased by 2.5 per cent from $40 000 to $41 000. It follows that the real income effect was responsible for a 2.5 per cent increase in consumption of housing and the substitution effect for the balance of 7.5 per cent. Accordingly, the compensating variation can be calculated as:

$$CV = (100 \times 10) + (10 \times 7.5 \times 0.5) = \$1037.50$$

Now let us calculate the EV. In this case, we have to put the household in its real income position after the price fall and ask what amount would compensate it for not having the lower price. Given that the substitution effect accounts for 7.5m^2 of the increase in consumption due to the subsidy, to retain the same real income as with the subsidy, the household would consume 102.50 units. Thus the equivalent variation would be:

$$EV = (102.5 \times 10) + (10 \times 7.5 \times 0.5) + \$1062.50$$

As predicted, with a fall in prices, CV < ΔCS < EV. However, even when expenditure is 25 per cent of total income and the price change is 10 per cent, the differences are small. CS is only 1.0 per cent greater than CV and 1.0 per cent smaller than EV. The percentage differences would be larger if expenditure were a higher percentage of income or the income elasticity higher. Zerbe and Dively (1994, p. 113) provide a table showing how these percentage differences vary with the expenditure share of total income and income elasticities.

Box 6.4 shows how differences between CV, ΔCS and EV can be derived formally from a utility function. This demonstrates the rigour of the approach. In general CV or EV values can be estimated by making plausible assumptions about the nature of the (income) utility function. However, because utility functions are not observed, the values derived from this approach may be open to question.

Box 6.4 Deriving consumer surplus, CV and EV from a utility function: an example

Say the consumer has the same utility function as in Box 6.1: $U(q_x,q_y)=q_x^{0.7},q_y^{0.3}$. Initially she faces prices (1, 1) and has an income of $100. Then the price of x increases to 2. What are the ΔCS, EV and CV?

First we need to derive the demand functions. In Box 6.1, we obtain:

$$Q_x = 0.7M/p_x \text{ and } q_y = 0.3M/p_y$$

Using this formula, consumer demand changes from (q_{x1},q_{y1}) = 70, 30 to (q_{x2},q_{y2}) = (35, 30).

Adopting a linear approximation of the demand curve, the loss of consumer surplus equals:

$$(35\times1) + (35\times1\times0.5) = 52.5$$

To calculate the CV we ask how much money would be necessary at prices (2, 1) to make the consumer as well off as she was when consuming the bundle (70, 30)? If the prices were (2, 1) and the consumer had income M, we can substitute into the demand functions to find that the consumer would optimally choose the bundle ($0.7M/2$, $0.3M$). Setting the utility of this bundle equal to the utility of the bundle (70, 30), we obtain:

$$(07M/2)^{0.7}(0.3M)^{0.3} = 70^{0.7}\times30^{0.3}$$

Solving for M gives us M = 162 (approximately). Hence to make the consumer as well off after the price rise as she was before it, she would need about 162 – 100 = $62 of additional income.

To calculate the EV we ask how much money would be necessary at price (1, 1) to make the consumer as well off as she would be consuming the bundle (35, 30). Letting M stand for this amount of money and following the same logic as before,

$$(07M/2)^{0.7}(0.3M)^{0.3} = 35^{0.7}\times30^{0.3}$$

This means that M = 62 approximately.

Thus if the consumer had an income $62 at the original prices, she would be just as well off as she would be facing the new prices and having an income of $100. The EV in income is therefore about 100 – 62 = 38.

As we predicted in Table 6.1, for a price rise,

$$CV > \Delta CS > EV$$

Concluding observations

Identifying, valuing and meeting individual preferences lies at the heart of public economics. In this chapter we have focused on the role of the ordinary demand curve because it is the foundation on which most economic valuations of individual preferences are based. These demand curves can be estimated for most market goods.

However, whether individuals should have to pay for goods and services or be compensated for not having them depends on views about individual rights. Given a decreasing marginal utility of income, WTA values are generally higher than WTP values though the difference is generally small for small changes in income. This means that it may sometimes be appropriate to estimate compensating or equivalent (CV) or (EV) values which are based on compensated demand curves.

Also, there are generally no observed demand curves for non-market goods. As we will see in Chapter 11, other valuation methods are often required to estimate the values of non-market goods. These include other revealed preference methods that analyse individual behaviour in various contexts to infer implicit valuations and stated preference methods that employ survey techniques to elicit the values of individual preferences. However, whichever valuation method is employed, it should be consistent with the valuation principles described in this chapter.

Summary

- The economic approach to public policy is based on valuations of individual preferences. These preferences are generally inferred from the choices that individuals make between various goods, including market and non-market goods.
- Formally, preferences can be represented by utility functions or indifference curves. Demand curves can be derived from utility functions in combination with budget constraints.
- Demand curves supply information on both the total value of a good and the marginal value of an extra unit of a good. Thus, estimates of demand curves or of relevant parts of demand curves are basic to estimates of the values of market goods.
- However, there are two kinds of demand curves. Ordinary (observed) demand curves include substitution and real income effects. Compensated demand curves show demand as a function of price holding real income constant.
- Valuations based on ordinary demand curves use the concept of consumer surplus.
- Valuations based on a compensated demand curve use the concepts of compensating or equivalent variation.
- The compensating variation (CV) value of a good is the maximum amount that may be taken from someone and leave them as well off with the good as without it. The equivalent (EV) value of a good is the minimum amount that someone will accept for not having the good and leave them as well off without the good as with it.
- Turning to losses of goods, the CV of a loss is the minimum amount that someone will accept as compensation and be as well off with the loss as without it. The EV of a loss is the maximum amount that someone will pay to stop the loss and be no worse off than with the loss.
- For most purposes, especially when income effects are small, changes in consumer surplus are a good measure of the value of individual preferences. However, when property rights are important and income effects are large, it may be important to estimate compensating or equivalent variation values.

Questions

1. According to Oscar Wilde, a cynic is a person who knows the price of everything and the value of nothing. Is this a good description of an economist?
2. What assumptions are necessary for supposing that individual preferences can be inferred and valued from the choices that individuals make?
3. Suppose that university fees increase by 20 per cent. How would the income and substitution effects contribute to the change in the quantity of university education demanded?
4. Explain why the marginal rate of substitution varies along a typical convex indifference curve. What is the implication for the relevant demand curve?
5. Economists often assume that for small changes in income, the income utility function can be regarded as linear. Is this a reasonable assumption? What are the implications of this assumption?
6. If Amy has a Cobb–Douglas utility function of the form $u(x, y) = q_x^{0.5} q_y^{0.5}$ where q_x and q_y are the quantities of two goods x and y, and the price of x is twice the price of y, what fraction of her income will she spend on goods x and y?
7. What is the difference between compensating and equivalent variations measures of value? When would it be appropriate to use one or other of these measures rather than a consumer surplus measure?
8. Can willingness to accept compensation values for losses of goods sometimes be observed in market transactions?
9. Ben has an income of $30 000 and spends $10 000 a year on housing. He rents 100 square metres at $100 per square metre per annum. The government now subsidises the rent by 20 per cent. Ben's income elasticity of demand for housing is 1.0 and his price elasticity of demand is –1.0. What is the consumer surplus, CV and EV value of the subsidy to Ben?
10. Amy has the same utility function as in Question 6. She has an income of $1000 and faces prices of $10 for each good x and y. The price of x increases to $20. What is the change in consumer surplus, CV, and EV value of this increase in price?
11. George Best, a famous Manchester United football player, reputedly said: 'I spent 90% of my money on booze, women and fast cars; the rest of my money I wasted'. Does this indicate bounded rationality?

Further Reading

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